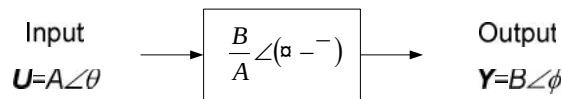
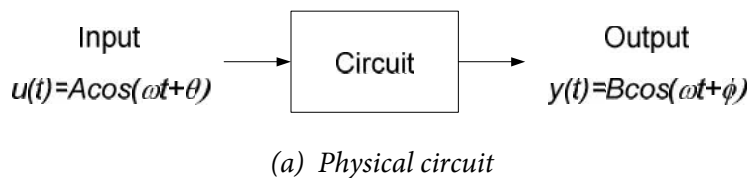


10.6.1: Passive RL Circuit Response

Overview:

In this lab assignment, we will be concerned with the steady-state response of electrical circuits to sinusoidal inputs. Figure 1(a) shows a block-diagram representation of the system. The input and output signals both have the same frequency, but the two signals can have different amplitudes and phase angles.

The analysis of the circuit of Figure 1(a) can be simplified by representing the sinusoidal signals as *phasors*. The phasors provide the amplitude and phase information of sinusoidal signals. By comparing the phasors representing the input and output signals, the effect of the circuit can be represented as an amplitude gain between the output and input signals and a phase difference between the output and input signals, as shown in Figure 1(b).



(b) Phasor representation of circuit input-output relationship.

Figure 1. Steady-state sinusoidal circuit analysis

In this lab assignment, we will measure the gain and phase responses of a passive RL circuit and compare these measurements with expectations based on analysis.

Before beginning this lab, you should be able to:

- Represent sinusoidal signals in phasor form
- Determine electrical circuit steady-state sinusoidal responses in phasor form

After completing this lab, you should be able to:

- Measure phasor form of circuit steady-state sinusoidal response
- Measure input impedance of electrical circuit

This lab exercise requires:

- Analog Discovery module
- Digilent Analog Parts Kit
- Digital multimeter (optional)

Symbol Key:

DEMO	Demonstrate circuit operation to teaching assistant; teaching assistant should initial lab notebook and grade sheet, indicating that circuit operation is acceptable.
ANALYSIS	Analysis; include principle results of analysis in laboratory report.
SIM	Numerical simulation (using PSPICE or MATLAB as indicated); include results of MATLAB numerical analysis and/or simulation in laboratory report.
DATA	Record data in your lab notebook.

General Discussion:

Consider the RL circuit shown in Figure 2 below. The input to the circuit is an applied voltage and we choose the current supplied by the source to be the system output. The differential equation relating the applied voltage $v_{IN}(t)$ to the input current $i_{IN}(t)$ can be obtained by applying KVL around the single loop:

If we assume that the input voltage and current are complex exponentials of the form:

$$v_{IN}(t) = Ve^{j(\omega t + \theta)} \quad (1)$$

$$i_{IN}(t) = Ie^{j(\omega t + \phi)} \quad (2)$$

We can write the circuit's input-output relation as a ratio between the current and the voltage:

$$\frac{\underline{I}}{\underline{V}} = \frac{Ie^{j\phi}}{Ve^{j\theta}} = \frac{1}{R + j\omega L} \quad (3)$$

Where $\underline{I}$ and $\underline{V}$ are *phasors* representing the magnitude and phase of the input current and input voltage to the circuit, respectively. This input-output relation can be written in terms of an amplitude gain and a phase shift:

$$\left| \frac{\underline{I}}{\underline{V}} \right| = \left| \frac{1}{R + j\omega L} \right| = \frac{1}{\sqrt{R^2 + \omega^2 L^2}} \quad (4)$$

$$\phi - \theta = -\tan^{-1}\left(\frac{\omega L}{R}\right) \quad (5)$$

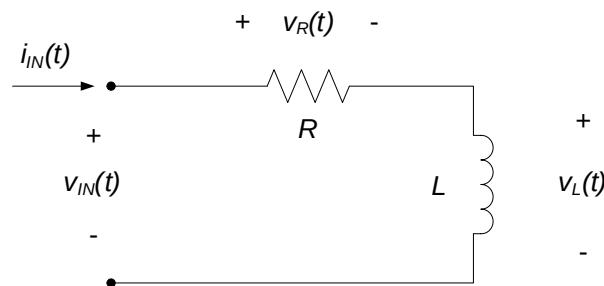


Figure 2. RL circuit.

Pre-lab:

- ANALYSIS** a. Show that the amplitude gain and phase difference between the input voltage and the input current are as shown in equations (4) and (5).
- ANALYSIS** b. The cutoff frequency for the circuit of Figure 2 is given to be $\omega_c = \frac{R}{L}$. Calculate the cutoff frequency for the circuit of Figure 2 if $L = 1\text{mH}$ and $R = 47\Omega$.
- ANALYSIS** c. Determine the gain and phase difference for the RL circuit for frequencies $\omega \approx 0$, $\omega \rightarrow \infty$, and $\omega = \omega_c$ if $L = 1\text{mH}$ and $R = 47\Omega$.
- ANALYSIS** d. Do your low and high frequency gain results in part (c) agree with your expectations based on the inductor's low and high frequency behavior? (e.g. calculate the inductor impedance at low and high frequencies, substitute these impedances into the circuit of Figure 2, calculate the response of the resulting resistive circuit, and compare to the results of part (c).)

Notes:

In this lab assignment, we will measure $v_{IN}(t)$ and $v_L(t)$. These measurements will be used to estimate the gain and phase difference between $v_{IN}(t)$ and $i_{IN}(t)$ and the gain and phase difference between $v_L(t)$ and $i_{IN}(t)$. These results will be compared with our expectations based on the pre-lab analyses. We do not have the ability to directly measure a time-varying current, so we will infer $i_{IN}(t)$ by measuring $v_{IN}(t) - v_L(t)$ and determining $i_{IN}(t)$ by:

$$i_{IN}(t) = \frac{v_{IN}(t) - v_L(t)}{R} \quad (6)$$

All signals we will be dealing with are sinusoidal. Appendix A of this lab assignment provides tips relative to gain and phase measurement of sinusoidal signals.

Lab Procedures:

Construct the circuit of Figure 2 with $L = 1\text{mH}$ and $R = 47\Omega$.

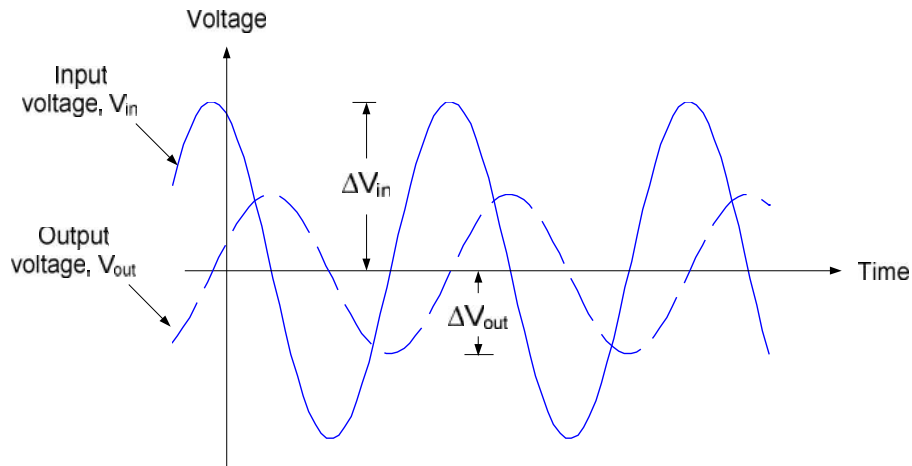
- DATA** a. Use your function generator to apply a sinusoidal input at $v_{IN}(t)$. Use your oscilloscope to display both $v_{IN}(t)$ and $v_L(t)$. Use the oscilloscope's math operation to display the input current, $i_{IN}(t)$, as provided by equation (6). Record the amplitude of $v_{IN}(t)$ and $i_{IN}(t)$ and the time delay between $v_{IN}(t)$ and $i_{IN}(t)$ for the following input voltage frequencies:
- $\omega \approx \omega_c / 10$ (low frequency input)
 - $\omega \approx 10\omega_c$ (high frequency input)
 - $\omega \approx \omega_c$ (corner frequency input)
- DEMO** b. Demonstrate operation of your circuit to the TA and have them initial the appropriate page(s) of your lab notebook and the lab worksheet.
- ANALYSIS** c. Calculate the measured gains and phase differences between $i_{IN}(t)$ and $v_{IN}(t)$ for the three frequencies listed in part (b) above. Compare your measured results with your expectations from the pre-lab. Comment on your results.

Appendix A – Measuring Gain and Phase:

The gain of a system at a particular frequency is the ratio of the magnitude of the output voltage to the magnitude of the input voltage at that frequency, so that:

$$\text{Gain} = \frac{\Delta V_{out}}{\Delta V_{in}}$$

where ΔV_{out} and ΔV_{in} can be measured from the sinusoidal input and output voltages as shown in the figure below.



The phase of a system at a particular frequency is a measure of the time shift between the output and input voltage at that frequency, so that:

$$\text{Phase} = \frac{\Delta T}{T} \times 360^\circ$$

where ΔT and T can be measured from the sinusoidal input and output voltages as shown in the figure below.

