

10.4.1: Impedance

Overview:

In this lab assignment, we measure impedances of resistors, capacitors, and inductors. The measured values will be compared with our expectations based on analyses.

Before beginning this lab, you should be able to:

- Represent sinusoidal signals in phasor form

After completing this lab, you should be able to:

- Measure impedances of passive circuit elements

This lab exercise requires:

- Analog Discovery module
- Digilent Analog Parts Kit
- Digital multimeter (optional)

Symbol Key:



Demonstrate circuit operation to teaching assistant; teaching assistant should initial lab notebook and grade sheet, indicating that circuit operation is acceptable.



Analysis; include principle results of analysis in laboratory report.



Numerical simulation (using PSPICE or MATLAB as indicated); include results of MATLAB numerical analysis and/or simulation in laboratory report.



Record data in your lab notebook.

General Discussion:

The concept of impedance is only appropriate in terms of the steady-state response of a circuit to a sinusoidal input. Impedance is a complex number which provides the relationship between voltage and current phasors in the circuit. Specifically, the impedance Z is the ratio of the voltage phasor to the current phasor:

$$Z = \frac{\underline{V}}{\underline{I}} = \frac{Ie^{j\phi}}{Ve^{j\theta}} = \left| \frac{V}{I} \right| e^{j(\phi-\theta)} \quad (1)$$

where the voltage and current of interest, $v(t)$ and $i(t)$, are assumed to be complex exponentials of the form:

$$v(t) = Ve^{j(\omega t + \theta)} = \underline{V}e^{j\omega t} \quad (2)$$

$$i(t) = Ie^{j(\omega t + \phi)} = \underline{I}e^{j\omega t} \quad (3)$$

$\underline{I}$ and $\underline{V}$ are *phasors* representing the magnitude and phase of the current and voltage, respectively. Impedance is a very general concept which can be applied to any combination of voltage and current in a circuit. In this lab project, however, we will be interested only in the impedance of specific circuit elements: resistors, capacitors, and inductors.

In order to experimentally determine impedance, we must determine both voltage and current. Since oscilloscopes do not measure current, we will use the measured voltage across a known resistance in order to infer the current through the circuit element of interest. The appropriate circuit schematic is as shown in Figure 1.

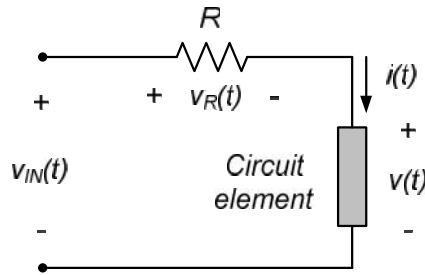


Figure 1. Circuit used for impedance measurements.

In the circuit of Figure 1, we can measure the voltages $v_R(t)$ and $v(t)$. The current through the circuit element of interest can be estimated from Ohm's law as:

$$i(t) = \frac{v_R(t)}{R} \quad (4)$$

By measuring the voltage $v(t)$ and estimating the current $i(t)$ for the circuit element in Figure 1, we can determine the circuit element's impedance from equation (1).

Pre-lab:

ANALYSIS

Assume that the voltages $v_R(t)$ and $v(t)$ in Figures 2 below are of the form:

$$v_R(t) = V_R \cos(\omega t + \theta)$$

$$v(t) = V \cos(\omega t + \phi)$$

Determine the impedances of the resistor R , the inductor L , and the capacitor C in Figure 2 below in terms of the phasor representations of the voltages $v_R(t)$ and $v(t)$. Express your results in terms of the magnitudes and phase angles of $v_R(t)$ and $v(t)$.

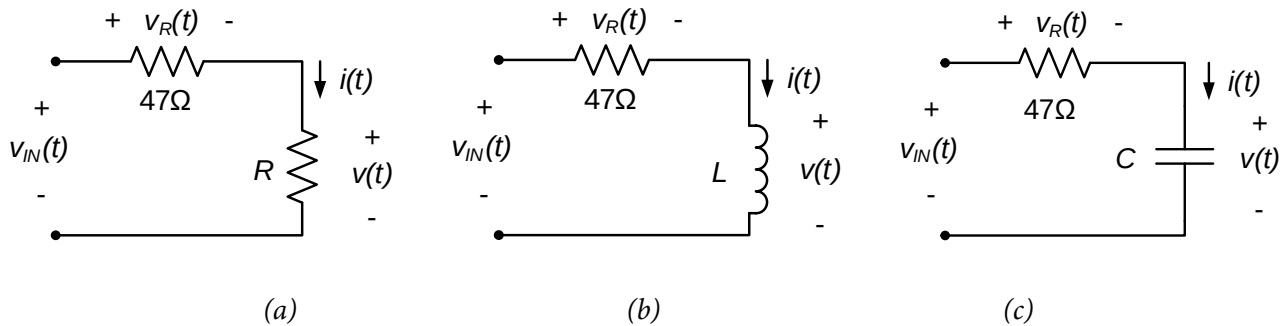


Figure 2. Circuits used in this lab project.

Lab Procedures:

- a. Construct the circuit of Figure 2(a) with $R = 100\Omega$.

- i. Use your function generator to apply a sinusoidal input voltage $v_{IN}(t)$ with an amplitude of 2V and a 0V offset. Use your oscilloscope to measure the voltages $v_R(t)$ and $v(t)$. Set up a math channel to display the current $i(t)$, according to equation (4). Record an image of the oscilloscope window, showing the signals $v_R(t)$, $v(t)$, and $i(t)$ for input signals with the following frequencies:

1. 1kHz
2. 5kHz
3. 10kHz

DATA

- ii. For each of the above three frequencies, tabulate:

- the amplitudes of $v(t)$ and $i(t)$, and
- the time difference between $v(t)$ and $i(t)$.

DATA

ANALYSIS

- iii. Calculate the impedance of the resistor at the above three frequencies. Compare your results to your expectations from the pre-lab analyses. Include a percent difference between your expectations and your measured impedances. **Note: Appendix A of this lab assignment provides tips relative to gain and phase measurement of sinusoidal signals.**

DEMO

- iv. Demonstrate operation of your circuit to the TA and have them initial the appropriate pages of your lab notebook and the lab worksheet.

b. Construct the circuit of Figure 2(b) with $L = 1\text{mH}$.

- i. Use your function generator to apply a sinusoidal input voltage $v_{IN}(t)$ with an amplitude of 2V and a 0V offset. Use your oscilloscope to measure the voltages $v_R(t)$ and $v(t)$. Set up a math channel to display the current $i(t)$, according to equation (4). Record an image of the oscilloscope window, showing the signals $v_R(t)$, $v(t)$, and $i(t)$ for input signals with the following frequencies:

4. 1kHz
5. 5kHz
6. 10kHz

DATA

DATA

- ii. For each of the above three frequencies, tabulate:
- the amplitudes of $v(t)$ and $i(t)$, and
 - the time difference between $v(t)$ and $i(t)$.

ANALYSIS

- iii. Calculate the impedance of the inductor at the above three frequencies. Compare your results to your expectations from the pre-lab analyses. Include a percent difference between your expectations and your measured impedances. **Note: Appendix A of this lab assignment provides tips relative to gain and phase measurement of sinusoidal signals.**

DEMO

- iv. Demonstrate operation of your circuit to the TA and have them initial the appropriate pages of your lab notebook and the lab worksheet.

c. Construct the circuit of Figure 2(c) with $C = 100\text{nF}$.

- i. Use your function generator to apply a sinusoidal input voltage $v_{IN}(t)$ with an amplitude of 2V and a 0V offset. Use your oscilloscope to measure the voltages $v_R(t)$ and $v(t)$. Set up a math channel to display the current $i(t)$, according to equation (4). Record an image of the oscilloscope window, showing the signals $v_R(t)$, $v(t)$, and $i(t)$ for input signals with the following frequencies:

7. 1kHz
8. 5kHz
9. 10kHz

DATA

DATA

- ii. For each of the above three frequencies, tabulate:
- the amplitudes of $v(t)$ and $i(t)$, and
 - the time difference between $v(t)$ and $i(t)$.

ANALYSIS

- iii. Calculate the impedance of the capacitor at the above three frequencies. Compare your results to your expectations from the pre-lab analyses. Include a percent difference between your expectations and your measured impedances. **Note: Appendix A of this lab assignment provides tips relative to gain and phase measurement of sinusoidal signals.**

DEMO

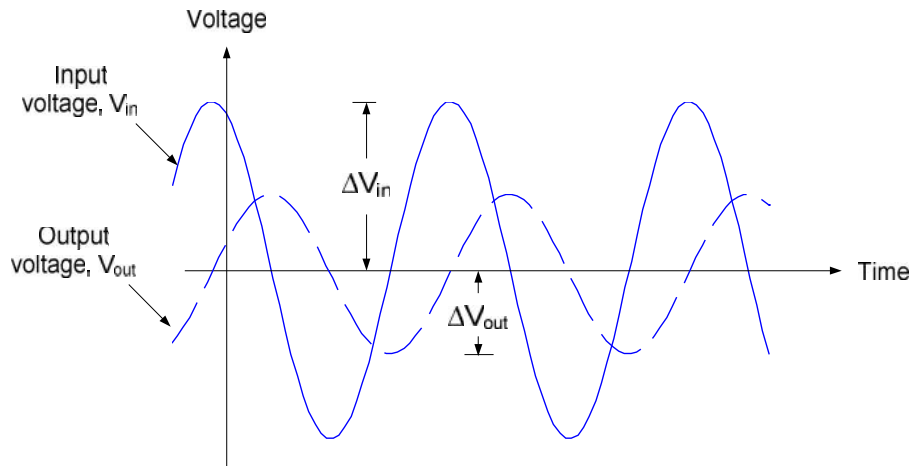
- iv. Demonstrate operation of your circuit to the TA and have them initial the appropriate pages of your lab notebook and the lab worksheet.

Appendix A – Measuring Gain and Phase:

The gain of a system at a particular frequency is the ratio of the magnitude of the output voltage to the magnitude of the input voltage at that frequency, so that:

$$\text{Gain} = \frac{\Delta V_{out}}{\Delta V_{in}}$$

where ΔV_{out} and ΔV_{in} can be measured from the sinusoidal input and output voltages as shown in the figure below.



The phase of a system at a particular frequency is a measure of the time shift between the output and input voltage at that frequency, so that:

$$\text{Phase} = \frac{\Delta T}{T} \times 360^\circ$$

where ΔT and T can be measured from the sinusoidal input and output voltages as shown in the figure below.

